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Optimization of welded square cellular plates with two different kinds of stiffeners

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Abstract: Cellular plates are constructed from two face plates and a stiffener grid welded between them. It is shown that the square cellular plates can be calculated as isotropic ones. Therefore, the classic formulae for maximum bending moment and deflection, valid for isotropic plates, can be used. The stiffeners can be made from halved rolled I-profiles, or from welded T-sections. These two kinds of cellular plates are optimized, and their minimum volumes and costs are compared to each other. The comparison shows that the cellular plate with welded T-stiffeners is more economic.

Keywords: cellular plates, welded stiffened plates, fabrication cost calculation, economy of welded structures, structural optimization

1. Introduction

Stiffened plate is one of the most frequently used structural components in welded structures. They are used as load-carrying elements of ships, bridges, offshore platforms, roofs, etc. In stability problems of welded structures the effect of initial imperfections and residual welding stresses should be taken into account. Many papers have been dealt with the stability and calculation of this kind of structures [1,2,3,4].

Two types of stiffened plates can be constructed: plate stiffened on one side (in the following briefly *stiffened plate*) and cellular plate. Cellular plates consist of two face plates and a grid of stiffeners welded between them. The cells produce a large torsional stiffness; thus, the cellular plates can be calculated as isotropic ones. Cellular plates have some advantages over stiffened ones as follows. (a) their torsional stiffness contributes to the overall buckling strength significantly, therefore, their dimensions (height and thickness) can be smaller, (b) their symmetry eliminates the large residual welding distortions, which can occur in stiffened plates due to shrinkage of eccentric welds. Therefore cellular plates can be cheaper than stiffened ones, as it will be shown in next section.

In their previous studies, the authors have designed cellular plates with halved rolled I-stiffeners [5,6,7]. In the present study, these rolled stiffeners are replaced by welded T-stiffeners. The comparison of the cellular plates with the two different kinds of stiffeners shows that using welded T-stiffeners significant savings in mass and cost can be achieved.

The formulae for the two kinds of stiffeners are nearly the same; thus, the formulae for halved rolled I-stiffeners are described and then the differences for welded T-stiffeners are given.

2. Bending and torsional stiffness of a cellular plate

The Huber’s equation for orthotropic plates in the case of a uniform transverse load *p*

$$B_x w'''' + 2Hw''\cdot\cdot + B_y w\cdot\cdot\cdot\cdot = p \tag{1}$$

where the torsional stiffness of an orthotropic plate is

$$H = B_{xy} + B_{yx} + \frac{\nu}{2}\left(B_x + B_y\right) \tag{2}$$

v is the Poisson ratio, *w* is the deflection

The corresponding bending and torsional stiffnesses are defined as

$$B_x = \frac{E_1 I_y}{a_y}; B_y = \frac{E_1 I_x}{a_x}; E_1 = \frac{E}{1-\nu^2} \tag{3}$$

for cellular plates

$$B_{xy} = \frac{GI_y}{a_y}; B_{yx} = \frac{GI_x}{a_x}; G = \frac{E}{2(1+\nu)} \tag{4}$$

where E is the Young modulus, G is the shear modulus,

I_x and I_y are the second moment of inertias in two directions,

a_x and a_y are the distances between stiffeners in two directions.

$$H = B_{xy} + B_{yx} + \frac{\nu}{2}\left(B_x + B_y\right) = \frac{E_1}{2}\left(\frac{I_y}{a_y} + \frac{I_x}{a_x}\right) \tag{5}$$

for plates of quadratic symmetry

$$H = B_x = B_y \tag{6}$$

Thus, the torsional stiffness of a cellular plate of quadratic symmetry equals to its bending stiffness.

3. Bending moments and deflections

Lee et al. [8] have solved the differential equation for rectangular orthotropic plates (Eq.1) supported at four corners by using a polynomial function.

They gave formulae for bending moments and deflections as a function of bending and torsional stiffnesses. In the case of a square cellular plate, the bending stiffnesses are equal to the torsional stiffness ($B_x = B_y = H$) and the maximum bending moment is

$$M_{max} = 0.15pL^2 \tag{7}$$

and the maximum deflection is expressed by

$$w_{max} = 0.025p_0L^4/B_x \tag{8}$$

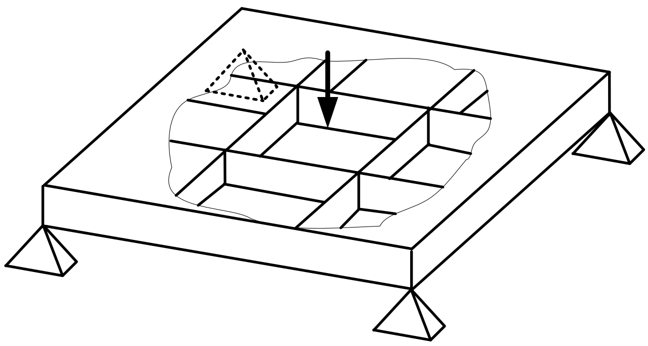

Figure 1. A cellular plate supported at four corners

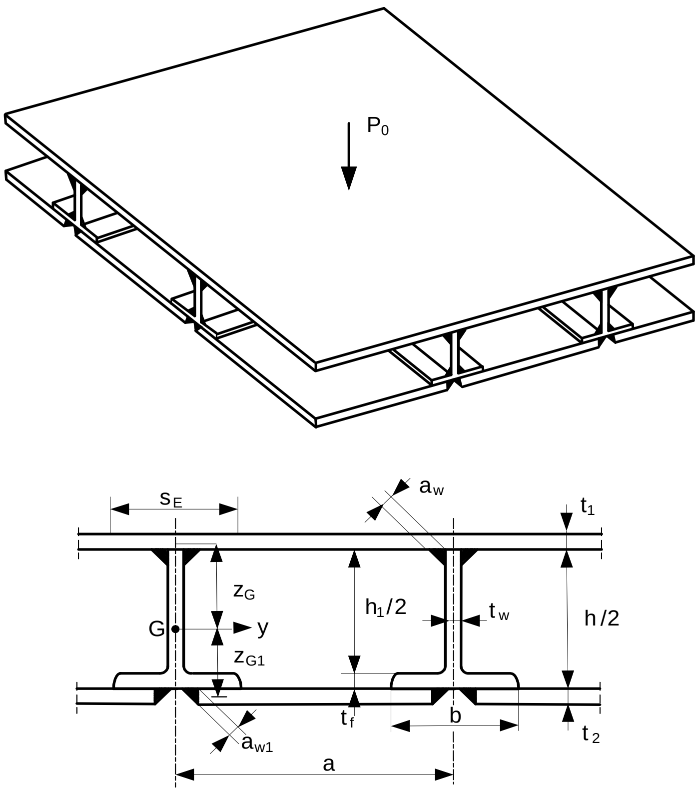

Figure 2. Cellular plate and dimensions of halved rolled I-section stiffener

where L is the plate edge length, p_0 is the factored intensity of the uniformly distributed normal load and p is the load intensity including the self mass of the plate.

Results for square isotropic plates according to Timoshenko & Woinowsky-Krieger [9]
for $\nu = 0.3$ (they used a simple approximate energy model)

$$M_{max} = 0.1404pL^2 \tag{9}$$

and

$$w_{max} = 0.0249p_0L^4/B_x \tag{10}$$

It can be seen that the constants are nearly the same.

4. Cellular plate with halved rolled I-section stiffeners

4.1 Geometric characteristics (Fig. 2)

The upper face plate parts can locally buckle from the compression stresses caused by bending. This local buckling is avoided by using effective plate widths according to Eurocode 3 [10]

$$s_e = \rho s, s = \frac{a}{n} \tag{11}$$

$$\rho = \frac{\lambda_p - 0.22}{\lambda_p^2}, \lambda_p = \frac{s}{56.8 \varepsilon t_1}, \varepsilon = \sqrt{\frac{235}{f_y}} \tag{12}$$

n is the number of spacing, f_y is the yield stress.

Cross-sectional area of a halved rolled I-section stiffener

$$A_S = \frac{h_1 t_w}{2} + b t_f, \quad h_1 = h - 2t_f \tag{13}$$

Cross-sectional area of a stiffener with upper and bottom base plate parts

$$A = s_e t_1 + a t_2 + A_S, \quad a = \frac{L}{n+1} \tag{14}$$

Distances of the centre of gravity

$$z_G = \frac{1}{A}\left[a t_2 \left(\frac{h}{2} + \frac{t_1}{2} + \frac{t_2}{2} \right) + b t_f \left(\frac{h_1 + t_1 + t_f}{2} \right) + \frac{h_1 t_w}{2} \left(\frac{h_1}{4} + \frac{t_1}{2} \right) \right] \tag{15}$$

$$z_{G1} = \frac{h + t_1 + t_2}{2} - z_G \tag{16}$$

Moment of inertia

$$I_y = s_E t_1 z_G^2 + a t_2 z_{G1}^2 + b t_f \left(\frac{h_1 + t_1 + t_f}{2} - z_G \right)^2 + I_{y1} \tag{17}$$

$$I_{y1} = \frac{h_1^3 t_w}{96} + \frac{h_1 t_w}{2} \left(\frac{h_1}{4} + \frac{t_1}{2} - z_G \right)^2 \tag{18}$$

Bending stiffness

$$B_x = \frac{E_1 I_y}{a}, E_1 = \frac{E}{1 - \nu^2} \tag{19}$$

Structural volumes corresponding to each fabrication phase are as follows:

$$V_1 = L^2 t_1, V_2 = V_2 + (n+2) A_S L, V_3 = V_2 + (n+2) A_S L \tag{20}$$

$$V_4 = V_3 + L^2 t_2 \tag{21}$$

Load intensity including the self mass

$$p = p_0 + \frac{\rho_0 V_4}{L^2} \tag{22}$$

4.2 Design constraints

Stress constraint including normal stress due to local bending of an upper base plate part with built-in edges according to Timoshenko & Woinowsky-Krieger [9]

$$\sigma_P = 0.0513 \frac{p_0 a^2}{t_1^2 / 6} = 0.3078 \frac{p_0 a^2}{t_1^2} \tag{23}$$

$$\sigma_2 = \frac{0.15 p L^2 z_G}{I_y} + \sigma_P \le \frac{f_y}{1.1} \tag{24}$$

Constraint on stress in the lower face plate

$$\sigma_1 = \frac{0.15 p L^2 z_{G1}}{I_y} \le \frac{f_y}{1.1} \tag{25}$$

Deflection constraint

$$w_{max} = \frac{0.025 p_0 L^4}{B_x} \le w_{allow} = \frac{L}{300} \tag{26}$$

Shear stress constraint at the corners

$$\tau = \frac{pL^2}{4 h_1 t_w} \le \frac{f_y}{1.1\sqrt{3}} \tag{27}$$

4.3 Fabrication constraints

Thickness limitation: t_{min} = 4 mm.
Limitation of the distance between stiffener flanges to allow the welding of the stiffener web to the upper base plate:

$$a - b \ge 300 \text{ mm.} \tag{28}$$

4.4 Structural characteristics to be changed (variables)

- number of stiffeners in one direction (square symmetry) n,
- thicknesses of the upper and bottom base plates t_1 and t_2,

- height of the rolled I-section stiffener *h*.

Dimensions of UB profiles are given in Table 1. (ArcelorMittal Profile Catalogue [11])

Table 1 Selected UB profiles according to the ArcelorMittal catalogue

UB Profile	h Mm	b mm	t_w mm	t_f mm	A_S mm^2	I_y x10^{-4} mm^4
152x89x16	152.4	88.7	4.5	7.7	2032	834
168x102x19	177.8	101.2	4.8	7.9	2426	1356
203x133x25	203.2	133.2	5.7	7.8	3187	2340
254x102x25	257.2	101.9	6.0	8.4	3204	3415
305x102x28	308.7	101.8	6.0	8.8	3588	5366
356x127x39	353.4	126.0	6.6	10.7	4977	10172
406x140x46	403.2	142.2	6.8	11.2	5864	15685
457x152x60	454.6	152.9	8.1	13.3	7623	25500
533x210x92	533.1	209.3	10.1	15.6	11740	55230
610x229x113	607.6	228.2	11.1	17.3	14390	87320
686x254x140	683.5	253.7	12.4	19.0	17840	136300
762x267x173	762.2	266.7	14.3	21.6	22040	205300
838x292x194	840.7	292.4	14.7	21.7	24680	279200
914x305x224	910.4	304.1	15.9	23.9	28560	376400
1016x305x349	1008.1	302	21.1	40.0	44420	722300
1016x305x393	1016.0	303	24.4	43.9	50020	807700

4.5 Numerical data

Plate edge length: L = 18 m, factored load intensity p_0 = 150 kg/m^2 = 0.0015 N/mm^2, yield stress of steel f_y = 355 MPa, elastic modulus E = 2.1x10^5 MPa, Poisson ratio ν = 0.3, steel density ρ = 7.85x10^{-6} kg/mm^3, ρ_0 = 7.85x10^{-5} N/mm^3.

4.6 Cost function

The cost function is formulated according to the fabrication sequence [12].

(a) Welding of the upper base plate (18x18 m) from 36 pieces of size 6 m x 1.5 m using single or double bevel welds with complete joint penetration (GMAW-C gas metal arc welding with CO_2):

$$K_{w1} = k_w\left[\Theta\sqrt{36\rho V_1} + 1.3C_1t_1^{n1}13L\right] \tag{29}$$

welding cost factor k_w = 1 \$/min, factor for the complexity of assembly Θ = 3,

for t_1 < 15 mm C_1 = 0.1939 and $n1$ = 2 (30a)

for t_1 > 15 mm C_1 = 0.1496 and $n1$ = 1.9029. (30b)

Note that the cost factor k_w varies between 0.5 and 2 \$/min for different countries and manufacturing companies, an average value of 1 \$/min is used.

(b) Welding of n+2 continuous stiffeners to the upper base plate by double fillet welds (GMAW-C)

$$K_{w2} = k_w \left[\Theta \sqrt{(n+3)\rho V_2} + 1.3x0.3394x10^{-3} a_w^2 2(n+2)L \right] \quad (31)$$

$a_w = 0.4t_w$, but $a_{wmin} = 4$ mm.

(c) Welding of n+2 intermittent stiffeners to the upper base plate and to the continuous stiffeners (webs with fillet welds, flanges with butt welds GMAW-C)

$$K_{w3} = k_w \left[\Theta \sqrt{(n^2+3n+3)\rho V_3} + T_1 + T_2 \right] \quad (32)$$

$$T_1 = 1.3x0.3394x10^{-3} a_w^2 (h_1 + b) 2(n+1)(n+2) \quad (33)$$

$$T_2 = 1.3 C_1 t_f^{n1} 2b(n+1)(n+2) \quad (34)$$

(d) Welding of the bottom plate parts to the flanges of stiffeners by fillet welds (GMAW-C)

$$K_{w4} = k_w \left[\Theta \sqrt{(n^2+2n+2)\rho V_4} + 1.3x0.3394x10^{-3} a_{w1}^2 4L(n+1) \right] \quad (35)$$

$a_{w1} = 0.4t_2$, but $a_{w1min} = 3$ mm.

Cost of material

$$K_M = k_M \rho V_4, \quad k_M = 1 \text{ \$/kg}, \quad (36)$$

For k_M the average value of 1 $/kg is used.

Cost of painting

$$K_P = k_P \Theta_P S_P, \Theta_P = 3, \quad k_P = 14.4x10^{-6} \text{ \$/mm}^2 \quad (37)$$

surface to be painted

$$S_P = 3L^2 + 2L(h_1 + b)(n+2) \quad (38)$$

The value of k_P is given from literature.

Total cost

$$K = K_M + K_{w1} + K_{w2} + K_{w3} + K_{w4} + K_P \quad (39)$$

4.7 Optimization and results

A systematic search (evaluation of every combination between the realistic ranges of unknowns) for the optima is performed using a MathCAD algorithm. The results are given in Table 2.

Table 2. Results of a systematic search. Halved rolled I-stiffeners. The optima are marked by bold letters. Allowed normal stress (σ_2) 322 MPa, allowed deflection w_{max} = 60 mm. Dimensions in mm

h	n	t_1	t_2	σ_2 MPa	w_{max} mm	Vx10^{-9} mm^3	Kx10^{-5} $
683.5	4	11	4	286	38	6.766	1.273
	5	9	4	315	40	6.435	1.258
	6	8	4	317	41	6.429	1.286
607.6	3	13	4	311	49	6.791	1.253
	4	**11**	**4**	316	50	6.400	**1.224**
	5	10	4	302	50	6.332	1.240
	6	9	4	302	51	6.265	1.258
	7	**8**	**4**	313	52	**6.197**	1.277
533.1	3	14	5	312	55	7.200	1.292
	4	12	5	314	57	6.761	1.255
	5	11	5	298	57	6.646	1.263
	6	10	5	296	58	6.530	1.272

5. Cellular plate with welded T-stiffeners

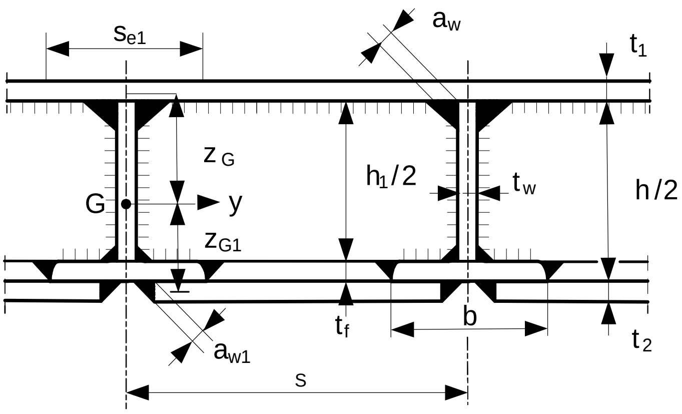

Figure 3. Cellular plate with welded T-stiffeners

5.1 Geometric characteristics (Fig.3)

The formulae of the characteristics of halved rolled I-section stiffeners are valid also for welded T-stiffeners with two differences: b = 30 mm and $t_f = t_2$.

5.2 Buckling constraint for the stiffener web

The web thickness should be taken to fulfil the following buckling constraint: according to Eurocode 3-1-5 for plates with varying longitudinal stress

$$\frac{0.15 p L^2 z_G}{I_y} \le C_x f_y \tag{40}$$

where

$$C_x = 1 \text{ when } \bar{\lambda}_p \le 0.673 \tag{41a}$$

$$C_x = \frac{\bar{\lambda}_p - 0.055(3+\psi)}{\bar{\lambda}_p^2} \quad \text{when} \quad \bar{\lambda}_p \geq 0.673 \tag{41b}$$

$$\bar{\lambda}_p = \frac{h_1/2}{t_w}\frac{1}{28.4\varepsilon\sqrt{k_\sigma}} \tag{42}$$

for $-1 \leq \psi < 0$

$$k_\sigma = 7.81 - 6.29\psi + 9.78\psi^2 \tag{43}$$

$$\psi = -\frac{h_1/2 - z_G}{z_G} \tag{44}$$

5.3 Cost function

In the cost function, the following changes are considered. Instead of Eqs (31, 32) the following formulae are used

$$K_{w2} = k_w\left[\Theta\sqrt{(2n+5)\rho V_2} + 1.3x0.3394x10^{-3}a_w^2 4(n+2)L\right] \tag{45}$$

$$K_{w3} = k_w\left[\Theta\sqrt{(2n^2+6n+5)\rho V_3} + T_1 + T_2\right] \tag{46}$$

5.4 Optimization and results

The results of a systematic search are given in Table 3.

Table 3. Results of a systematic search. Welded T-stiffeners. The optima are marked by bold letters. Allowed normal stress (σ_2) 322 MPa, allowed deflection w_{max} = 60 mm. Dimensions in mm

h	n	t_1	t_2	t_w	σ_1 MPa	w mm	web buckling MPa	Vx10^{-9} mm^3	Kx10^{-5} \$
1400	4	7	4	6	309	12	187<201	4.492	1.075
1300	**5**	**6**	**4**	**6**	320	16	204<215	**4.247**	1.074
1200	3	9	4	6	321	16	206<232	4.877	1.073
1100	**4**	**8**	**4**	**6**	309	20	215<251	4.622	**1.068**
1000	5	7	4	7	317	24	232<310	4.469	1.070
900	4	9	4	5	311	26	237<258	4.720	1.070
800	5	8	4	7	322	37	258<375	4.622	1.072

6. Comparison of the two optimized cellular plates with different stiffeners

It has been shown in previous studies that, in the case of square symmetry, the torsional stiffness of cellular plates equals to their bending stiffness. Thus, they can be calculated as isotropic ones and the bending moments and deflection for a square plate supported at four corners can be obtained by using the formulae for isotropic plates.

In the optimization process the four variables are as follows: height and number of halved rolled I-section stiffeners, or T-stiffener as well as the thicknesses of upper and bottom face plates. A systematic search considers the constraints on stresses and deflection as well as the cost function to be minimized.

According to the results summarized in Tables 2 and 3 it can be concluded that the cellular plate with welded T-stiffeners is more economic, than that with halved rolled I-section stiffeners, since the mass is (6.197-4.247)/6.197x100 = 31% smaller and the cost is (1.224-1.068)/1.224x100 = 12% smaller. It is also shown, that the optima for cost and mass minima are different. It can be up to 10% saving in mass (volume).

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